88117209

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

MATHEMATICS
HIGHER LEVEL
PAPER 3 – SETS, RELATIONS AND GROUPS

Friday 4 November 2011 (morning)

1 hour

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.

Please start each question on a new page. Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. In particular, solutions found from a graphic display calculator should be supported by suitable working, e.g. if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

1. *[Maximum mark: 20]*

(a) Consider the following Cayley table for the set $G = \{1, 3, 5, 7, 9, 11, 13, 15\}$ under the operation $\times_{16}$, where $\times_{16}$ denotes multiplication modulo 16.

$\times_{16}$	1	3	5	7	9	11	13	15
1	1	3	5	7	9	11	13	15
3	3	a	15	5	11	b	7	c
5	5	15	9	3	13	7	1	11
7	7	d	3	1	e	13	f	9
9	9	11	13	g	1	3	5	7
11	11	h	7	13	3	9	i	5
13	13	7	1	11	5	j	9	3
15	15	13	11	9	7	5	3	1

(i) Find the values of a, b, c, d, e, f, g, h, i and j.

(ii) Given that $\times_{16}$ is associative, show that the set G, together with the operation $\times_{16}$, forms a group. *[7 marks]*

(This question continues on the following page)

(Question 1 continued)

(b) The Cayley table for the set $H = \{e, a_1, a_2, a_3, b_1, b_2, b_3, b_4\}$ under the operation $*$, is shown below.

$*$	e	a_1	a_2	a_3	b_1	b_2	b_3	b_4
e	e	a_1	a_2	a_3	b_1	b_2	b_3	b_4
a_1	a_1	a_2	a_3	e	b_4	b_3	b_1	b_2
a_2	a_2	a_3	e	a_1	b_2	b_1	b_4	b_3
a_3	a_3	e	a_1	a_2	b_3	b_4	b_2	b_1
b_1	b_1	b_3	b_2	b_4	e	a_2	a_1	a_3
b_2	b_2	b_4	b_1	b_3	a_2	e	a_3	a_1
b_3	b_3	b_2	b_4	b_1	a_3	a_1	e	a_2
b_4	b_4	b_1	b_3	b_2	a_1	a_3	a_2	e

(i) Given that $*$ is associative, show that H together with the operation $*$ forms a group.

(ii) Find two subgroups of order 4. *[8 marks]*

(c) Show that $\{G, \times_{16}\}$ and $\{H, *\}$ are not isomorphic. *[2 marks]*

(d) Show that $\{H, *\}$ is not cyclic. *[3 marks]*

2. *[Maximum mark: 10]*

(a) Determine, using Venn diagrams, whether the following statements are true.

(i) $A' \cup B' = (A \cup B)'$

(ii) $(A \setminus B) \cup (B \setminus A) = (A \cup B) \setminus (A \cap B)$ *[6 marks]*

(b) Prove, without using a Venn diagram, that $A \setminus B$ and $B \setminus A$ are disjoint sets. *[4 marks]*

3. *[Maximum mark: 6]*

Show that the set, M, of matrices of the form $\begin{pmatrix} a & 0 \\ 0 & \frac{1}{a} \end{pmatrix}$, $a \in \mathbb{R}^+$, forms a group under matrix multiplication.

4. *[Maximum mark: 14]*

The group G has a subgroup H. The relation R is defined on G by xRy if and only if $xy^{-1} \in H$, for $x, y \in G$.

(a) Show that R is an equivalence relation. *[8 marks]*

(b) The Cayley table for G is shown below.

	e	a	a^2	b	ab	a^2b
e	e	a	a^2	b	ab	a^2b
a	a	a^2	e	ab	a^2b	b
a^2	a^2	e	a	a^2b	b	ab
b	b	a^2b	ab	e	a^2	a
ab	ab	b	a^2b	a	e	a^2
a^2b	a^2b	ab	b	a^2	a	e

The subgroup H is given as $H = \{e, a^2b\}$.

(i) Find the equivalence class with respect to R which contains ab.

(ii) Another equivalence relation ρ is defined on G by $x\rho y$ if and only if $x^{-1}y \in H$, for $x, y \in G$. Find the equivalence class with respect to ρ which contains ab. *[6 marks]*

5. *[Maximum mark: 10]*

Consider the functions $f : A \to B$ and $g : B \to C$.

(a) Show that if both f and g are injective, then $g \circ f$ is also injective. *[3 marks]*

(b) Show that if both f and g are surjective, then $g \circ f$ is also surjective. *[4 marks]*

(c) Show, using a single counter example, that both of the converses to the results in part (a) and part (b) are false. *[3 marks]*